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TECHNICAL NOTE 3779

TENTATIVE METHOD FOR CALCULATION OF THE SOUND FIELD ABOUT
A SOURCE OVER GROUND CONSIDERING DIFFRACTION

AND SCATTERING INTO SHADOW ZONES

By David C. Pridmore-Brown and Uno Ingard

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SUMMARY

A semiempirical method is given for the calculation of the sound field about a source over ground considering the effects of vertical temperature and wind gradients as well as scattering of sound by turbulence into shadow zones. The diffracted field in a wind-created shadow zone is analyzed theoretically in the two-dimensional case and is shown to be similar to the results obtained for a temperature-created shadow field as given in NACA TN 3494. The frequency and wind-velocity dependence of the scattered field into the shadow zone is estimated from Lighthill's theory, and on the basis of these two field contributions a semiempirical formula is constructed for the total field which contains two adjustable parameters. From this expression a set of charts has been prepared showing equal sound-pressure contours at 10 feet above ground for various source heights, wind velocities, and frequencies.

The two adjustable parameters in the formula were obtained from measurements using a relatively small source height (10 feet). The parameters should actually be a function of height determined by the wind and temperature profiles. However, in these preliminary calculations the parameters have been kept constant, and the fields, particularly for large source heights, must be considered as preliminary estimates to be corrected when more information is available.

INTRODUCTION

As is well known, sound energy follows curved paths in the presence of a temperature gradient. Theory indicates that near a sound source this phenomenon has only a slight effect on the intensity field, the correction being of the second order in the temperature gradient. However, if a sound source is placed above ground in a negative temperature gradient (lapse rate), a shadow zone will form at a certain distance from it because the rays are screened off by the ground from entering this region. The same phenomenon occurs in the presence of a wind

gradient, a shadow being formed on the upwind side of a source if the wind velocity increases with height.

Previous reports on the present project in atmospheric acoustics have described two sets of measurements, one performed in the laboratory (ref. 1) and the other in the field (results as yet unpublished). In the laboratory measurements a uniform (static) temperature gradient was set up in a two-dimensional propagation chamber. Below a certain frequency this chamber allowed cylindrical waves to propagate between two parallel steel plates spaced a distance apart less than half a wavelength. The temperature gradient was high enough to secure a sound shadow within a short distance from a source placed near one end. The measured shadow field in the chamber agreed well with diffraction theory, which predicts that the sound pressure in this region should fall off exponentially with distance from the shadow boundary at a rate which increases with the frequency and the temperature gradient. Finally, the two-dimensional analysis of the diffracted field in a wind-created shadow, which is included in the appendix, predicts the same behavior of the sound field as was predicted for the temperature-created shadow.

Measurements in the field taken in a wind-created shadow gave results of the type illustrated qualitatively in figure 1. The sound intensity at a point within the shadow was observed first to diminish with frequency in accordance with diffraction theory, but after a certain frequency it increased again, thus indicating the presence of another mode of energy penetration into the shadow which predominated over diffraction at the higher frequencies. It seems reasonable to suppose that this contribution to the higher frequencies is due to the presence of turbulence in the atmosphere which scatters sound from the normal zone into the shadow as illustrated schematically in figure 2.

On the basis of this mechanism a semiempirical theory is given which attempts to explain the field measurements and to produce an approximate method by which the sound field can be predicted.

The present investigation was conducted at the Acoustics Laboratory of the Massachusetts Institute of Technology under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.

SYMBOLS

A,B	empirical constants
c	speed of sound, fps
f	frequency, cps

H	relative humidity, percent
h	source height, ft
I	sound intensity
k	propagation constant, $2\pi f/c$
p	sound pressure field
R	source-to-receiver distance, ft
R_0	distance between source at height h and receiver at height 10 ft, ft
r	radial coordinate, distance from source, ft
r_0	horizontal component of R_0 , ft
s	wind-velocity gradient, $\frac{1}{c} \frac{dv}{dz}$
t	time, sec
v	wind velocity, fps unless otherwise indicated
x	horizontal distance from source, ft
x_0	horizontal distance from source to shadow boundary, ft
z	vertical coordinate, receiver height, ft
α	propagation factor
$\beta = v/c$	
τ	temperature gradient, $^{\circ}\text{C}/\text{ft}$
ϕ	angle between wind and sound direction; velocity potential
$\omega = 2\pi f$	

ANALYSIS

Diffracted Field

A two-dimensional calculation of the sound field from a source located in a wind gradient, given in the appendix, predicts that within

the acoustic shadow on the upwind side of the source the pressure field due to diffraction alone should be of the form

$$p \sim \exp \left[-B' (x - x_0) k^{1/3} s^{2/3} \right]$$

where x_0 is the horizontal distance to the shadow-zone boundary, $k = 2\pi f/c$, $s = (1/c)(dv/dz)$, and $B' = 0.7$ for a hard boundary. In three dimensions it seems probable that the same relation will hold in the direction against the wind with the addition of the factor $1/\sqrt{r}$ to take account of divergence. Moreover, in practice it turns out that the wind velocity v and its gradient are roughly proportional. Substituting one for the other and at the same time replacing B' by B to allow for the unknown proportionality leads to the assumption for the diffracted sound in the shadow zone of an expression of the form

$$I_B \frac{r_0}{r} e^{-2B(r-r_0)} f^{1/3} v^{2/3}$$

where I_B is the sound intensity on the shadow boundary. It is assumed that I_B varies inversely as the square of the distance from the source, that is, that it has the form A/R_0^2 where R_0 is the distance from the source at height h to the shadow boundary at height z .

Scattered Part

In addition to the sound field diffracted into the shadow region there will also be a contribution from the scattering of sound from turbulence in the sound region, as already indicated schematically in figure 2. The theory of the scattering of sound from turbulence presented by Lighthill (ref. 2) is essentially applicable to only a small region of turbulence in space and has to be modified to a multiple-scattering theory in order to apply in the present case. However, without having performed such a detailed analysis, it is assumed that in this case the frequency and velocity dependence of the scattered energy will still be the same as that given by Lighthill. In particular, attention is limited to that region in which the sound wavelength can be considered small compared with the scale of the turbulence. In that case the scattered energy should be proportional to the sound frequency squared as well as to the mean square of the velocity fluctuations. Since the velocity fluctuations are proportional to the average wind velocity v , the scattered sound may therefore be set proportional to $f^2 v^2$. Although the scattered sound must originate from all over the sound zone, in this simple analysis it is assumed that this distributed source can be replaced by a single source located between the receiver and the shadow boundary.

It is also assumed that the sound intensity from this equivalent source spreads out in a spherical wave and accordingly may be written as proportional to $\left(\frac{fv}{R-a}\right)^2$ where R is the source-to-receiver distance. To insure that this equivalent sound source lies within the normal zone a is set equal to $\frac{1}{2} r_0$ where r_0 is the horizontal distance to the shadow boundary. The exact value assigned to a turns out to be unimportant since, for a large value of R , it will have a negligible effect on the value of the term, and for a small value of R this term is itself small compared with the one representing the diffracted sound.

Total Field

From the previous discussions the following expression is obtained for the total sound intensity within the acoustic shadow:

$$I = \left(\frac{fv}{R - \frac{1}{2} r_0}\right)^2 + A \frac{r_0}{r(R_0)^2} e^{-2B(r-r_0)} f^{1/3} v^{2/3} \quad (1)$$

The assumption is made here that the shadow distance r_0 is related to the source height h as it is for a linear wind gradient; namely, $r_0 = Kv^{-1/2}(1 + \sqrt{h/z})$ where K is a constant. The receiver height z is put equal to 10 feet throughout the calculations. From the definitions it follows also that $R^2 = r^2 + (z-h)^2$ and $R_0^2 = r_0^2 + (z-h)^2$. The unknown constants A and B can be found by fitting equation (1) to data obtained in field measurements.

The constant A is essentially a measure of the relative strengths of the diffracted and scattered sound pressure levels and for the purpose of this analysis is considered to be independent of the other physical quantities. Similarly, B is assumed to depend only on the ground impedance. The constants A and B can be estimated empirically from a knowledge of (1) the critical frequency at which the sound pressure level at a given point in the shadow is a minimum under a known set of physical conditions and (2) the total attenuation brought about by wind effects, which is measured under these same conditions. Now, in one set of field measurements (ref. 3) made under the following conditions

$r = 1,000$ ft upwind of source

$r_0 = 200$ ft

$v = 15$ fps

$h = 10$ ft

the critical frequency turned out to be about 1,000 cps. At this frequency a total attenuation of 37 decibels more than the value at the shadow boundary was measured; in other words, from equation (1),

$$10 \log \left\{ \frac{(2fv/r_0)^2 + (A/r_0^2)}{\left[fv / \left(r - \frac{1}{2} r_0 \right) \right]^2 + (A/r_0) e^{-2\gamma}} \right\} = 37 \quad (2)$$

where $\gamma = B(r - r_0)r^{1/3}v^{2/3}$.

Differentiation of equation (1) with respect to frequency gives A in terms of γ and critical frequency as

$$A = \frac{3}{\gamma} \left(\frac{fv}{r - \frac{1}{2} r_0} \right)^2 \frac{r(R_0)^2}{r_0} e^{2\gamma}$$

Substituting for A in equation (2) gives $\gamma = 4.1$ nepers whence

$$A \cong 50 \text{ ft}^{-2} \text{ sec}^{-4}$$

$$B = 8 \times 10^{-5} \text{ sec ft}^{-5/3}$$

Finally, the relation adopted between shadow distance and wind velocity $r_0 = Kv^{-1/2}(1 + \sqrt{h/z})$ determines K as

$$K = 400 \text{ ft}^{3/2} \text{ sec}^{-1/2}$$

NUMERICAL CALCULATIONS

Equation (2) giving the critical frequency is plotted in figure 3 against wind velocity for several values of the source-to-receiver parameter r and for a source height of 10 feet. These curves divide the upper right-hand region where scattering predominates from the lower left-hand region where diffraction is important. The sound intensity given in equation (1) in terms of distance, frequency, wind velocity, and source height has been plotted as a function of distance for several values of the other parameters. Figures 4(a) to 4(f) are

plots of the sound-pressure-level increment (or $10 \log (I/I_0)$ where $I_0 = \text{Constant}$) on the upwind side of a source located 10, 100, and 1,000 feet above ground and driven at 250 and 500 cps in various winds. From these plots polar diagrams giving the equal sound-level contours around the source were prepared under the following two assumptions: (1) Within the shadow, formula (1) was assumed to hold for an angle ϕ between wind and sound provided that v was replaced by $v \cos \phi$,¹ and (2) within the normal region, the sound intensity was assumed to fall off with the square of the distance from the source. These polar diagrams are given in figures 5(a) to 5(l) for wind velocities of 10 and 20 mph, frequencies of 250 and 500 cps, and source heights of 10, 100, and 1,000 feet making a total of 12 diagrams. In each of these the position of the shadow boundary has been drawn in as a dashed line. The decibel values labeling the equal sound-level contours indicate the decrease of sound pressure level below the reference sound pressure level as obtained in a free field at 100 feet from the source.

As a specific example of the use of these diagrams, consider a sound source such as a jet to be located 10 feet above ground in a 20-mph wind measured at 10 feet above ground. The problem is to predict the sound pressure level 10 feet above ground at a distance of 1,000 feet from the source. Assume the power output of the engine to be such that at 100 feet its sound pressure level is 120 decibels and assume that it is mainly concentrated around 250 cps. The numbers given in the charts indicate the number of decibels that has to be subtracted from the value at 100 feet in order to obtain the sound pressure level at the point in question. Therefore, in this example the sound pressure level on the downwind side should be approximately $120 - 20 = 100$ decibels at 1,000 feet, while on the upwind side it becomes $120 - 46 = 74$ decibels at that distance. In the direction 45° into the wind the level should be 80 decibels.

The formula presented in equation (1) can, of course, be used to prepare charts for more extended ranges of the variables considered. If, in addition to the wind gradient, there also exists a temperature gradient τ , the quantity v in equation (1) must be replaced by $v \cos \phi + 10^4 \tau$ (τ is the temperature gradient in $^\circ\text{C}$ per foot). Thus, if $v < 10^4$, at some angle ϕ given by $\cos \phi = -v \cos \phi / 10^4 \tau$ the effects of wind and temperature gradients cancel each other. This angle demarcates the boundary between the shadow and the normal zone. The charts presented herein refer to the situation when there is no temperature gradient ($\tau = 0^\circ/\text{ft}$ and $\phi = 90^\circ$).

¹A preliminary analysis of the three-dimensional wind equation indicates that this procedure is justified.

It should be noted that from the sound pressure level obtained here should be subtracted the effect of air absorption which can be written as approximately

$$\left(\frac{f}{1,000}\right)^{3/2} \frac{450}{20 + H} \text{ db/mile}$$

where H is the relative humidity in percent. This effect is negligible in most cases. For example, at 250 cps and 50-percent relative humidity it amounts to an attenuation of only 0.8 decibel in a mile while at 500 cps the attenuation is 2.3 decibels per mile.

Massachusetts Institute of Technology,
Cambridge, Mass., August 19, 1955.

APPENDIX

TWO-DIMENSIONAL ANALYSIS OF DIFFRACTED FIELD IN

WIND-CREATED SHADOW

Cylindrical Source in Wind Gradient

Consider a cylindrical source to be located a height h above ground in a horizontal wind. The cylinder is parallel to the ground and perpendicular to the wind direction. The sound field at large distances from such a source is sought as the solution to the wave equation in a moving medium (ref. 4) which is

$$\nabla^2 \phi = \frac{1}{c^2} \left(\frac{\partial}{\partial t} + \bar{v} \nabla \right)^2 \phi = 0$$

If the x -axis is taken in the wind direction and a time factor $e^{-i\omega t}$ is canceled the equation becomes approximately, for $\beta(z) = v/c \ll 1$,

$$(\nabla^2 + k^2) \phi + 2ik\beta \frac{\partial \phi}{\partial x} = 0$$

Two-dimensional solutions to this equation are of the form

$$F(z)e^{\alpha x}$$

where

$$\frac{d^2 F}{dz^2} + (\alpha^2 + k^2 + 2ik\beta\alpha)F = 0$$

For simplicity a linear wind gradient $\beta = -sz$ is assumed (s constant) for which

$$F(z) = w^{1/3} H_{1/3}^{(1)}(w)$$

where

$$w(z) = \frac{i(k^2 + \alpha^2 - 2iks\alpha z)^{3/2}}{3ks} \quad (A1)$$

and $H_{1/3}^{(1)}$ is the one-third order Hankel function of the first kind. If these solutions are required to satisfy the boundary condition $dF/dz = 0$ at $z = 0$ (corresponding to infinite ground impedance), an infinite set of functions $F_n(z)$ with corresponding propagation factors α_n is obtained. The sound field may be expressed as a sum of these functions in the form

$$\phi = e^{-i\omega t} \sum_n F_n(h) F_n(z) e^{\alpha_n x} \quad (A2)$$

The boundary condition $dF/dz = 0$ at $z = 0$ leads to

$$H_{-2/3}^{(1)}(w) = 0$$

whence,

$$\frac{i(k^2 + \alpha^2)^{3/2}}{ks} = B e^{-i\pi}$$

where $B = 0.685$ for the first root. The propagation factor α is then

$$\alpha = \pm ik \left[1 - (3sk^{-2}\alpha B)^{2/3} e^{i\pi/3} \right]^{1/2}$$

Here, in order to insure outgoing waves in conjunction with the time factor $e^{-i\omega t}$, one must use the plus sign to the right of the source and the minus sign to the left. Since (for high frequencies) the second term in the brackets is small compared with the first, one may put approximately $\alpha = \pm ik$ in this term and so obtain for α on

expanding (a) to the right of the source,

$$\alpha = ik \left[1 - \frac{1}{2} (3sk^{-1}B)^{2/3} e^{2i\pi/3} \right] \quad (A3)$$

and (b) to the left of the source,

$$\alpha = -ik \left[1 - \frac{1}{2} (3sk^{-1}B)^{2/3} \right] \quad (A4)$$

To the left of the source α is purely imaginary and leads to undamped propagation, while to the right it is complex, indicating that the waves are attenuated in this direction. This is to be expected from the form of the velocity function which leads to an effective refraction of the waves, downward to the left and upward to the right.

High-Frequency Behavior of Solution

The asymptotic form of each mode of the solution (A2) for high frequencies is (apart from a phase factor)

$$\phi \sim e^{i[w(z)+w(h)+\alpha x-\omega t]}$$

where $w(z)$ is given in equation (A1). If it is assumed that $\alpha = ik(1 - \delta)$, where from equation (A3) $\delta = \frac{1}{2} (3sk^{-1}B)^{2/3} e^{-2i\pi/3}$ on the upwind side of the source, then to the first order in δ and sz

$$w(z) \approx - \frac{[k^2 - k^2(1 - 2\delta) + 2k^2sz]^{3/2}}{3k^2s}$$

$$\frac{2\sqrt{2}}{3} ks^{1/2} z^{3/2} \left(1 + \frac{3}{2} \frac{\delta}{sz} \right)$$

Now, from ray theory the horizontal distance from the source at height h to the shadow boundary at height z is

$$x_0 = \sqrt{2} \left[(z/s)^{1/2} + (h/s)^{1/2} \right]$$

while the corresponding travel time of the ray is

$$t_0 = \frac{\sqrt{2}}{c} \left[\left(\frac{z}{s} \right)^{1/2} \left(1 + \frac{2}{3} sz \right) + \left(\frac{h}{s} \right)^{1/2} \left(1 + \frac{2}{3} sh \right) \right]$$

Rewriting w in terms of these parameters gives

$$w(z) + w(h) = k(x_0 - ct_0) - kx_0\delta$$

The high-frequency behavior of the solution is then

$$\phi \sim e^{-i \left[w(t-t_0) - k(x-x_0)(1-\delta) \right]}$$

Within the shadow zone ($x > x_0$) the solution is essentially represented by the first mode, which is attenuated at a rate given by

$$k \operatorname{Im} \delta = \frac{\sqrt{3}}{4} (3Bs)^{2/3} k^{1/3} \\ \approx 0.7 s^{2/3} k^{1/3} \text{ nepers/distance}$$

where Im signifies the imaginary part.

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1. Pridmore-Brown, David C., and Ingard, Uno: Sound Propagation Into the Shadow Zone in a Temperature-Stratified Atmosphere Above a Plane Boundary. NACA TN 3494, 1955.
2. Lighthill, M. J.: On the Energy Scattered From the Interaction of Turbulence With Sound or Shock Waves. Proc. Cambridge Phil. Soc., vol. 49, pt. 3, July 1953, pp. 531-551.
3. Ingard, Uno: The Physics of Outdoor Sound. Proc. Fourth Annual Nat. Noise Abatement Symposium, vol. 4, 1953.
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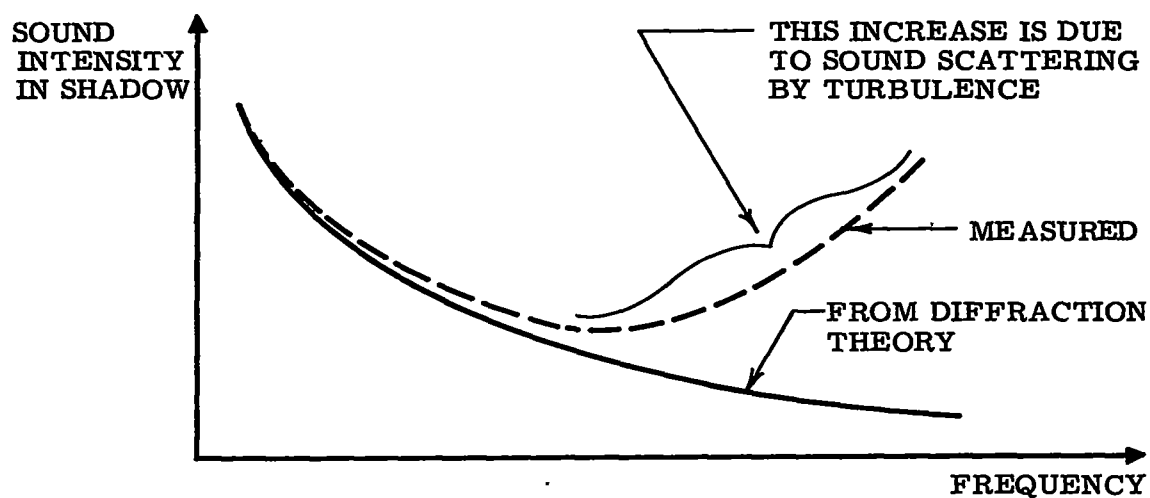


Figure 1.- An illustrative plot of frequency spectrum of sound intensity in shadow showing departure from diffraction theory.

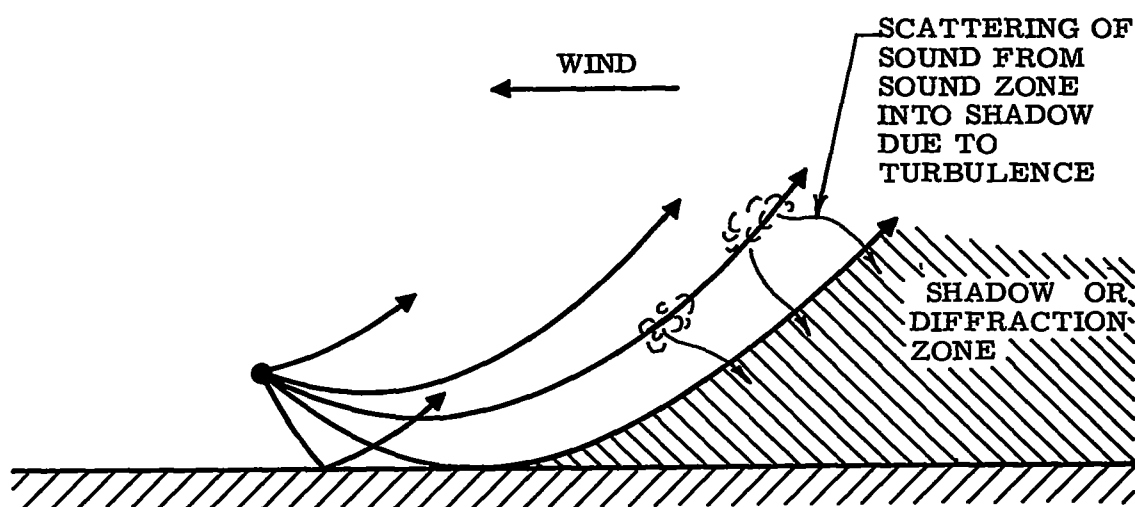


Figure 2.- Schematic diagram illustrating how sound penetrates shadow owing to the presence of turbulent scattering in sound zone.

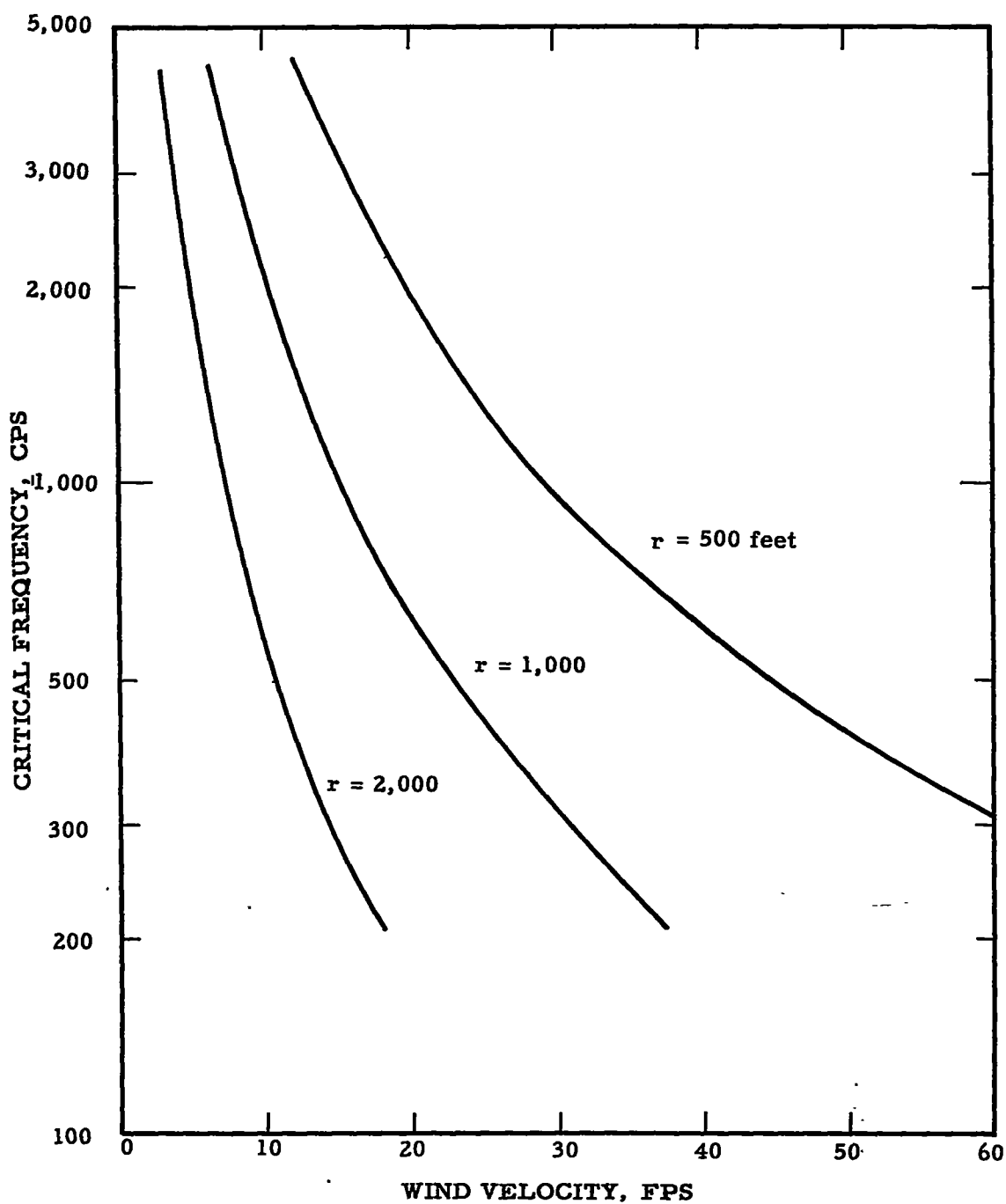
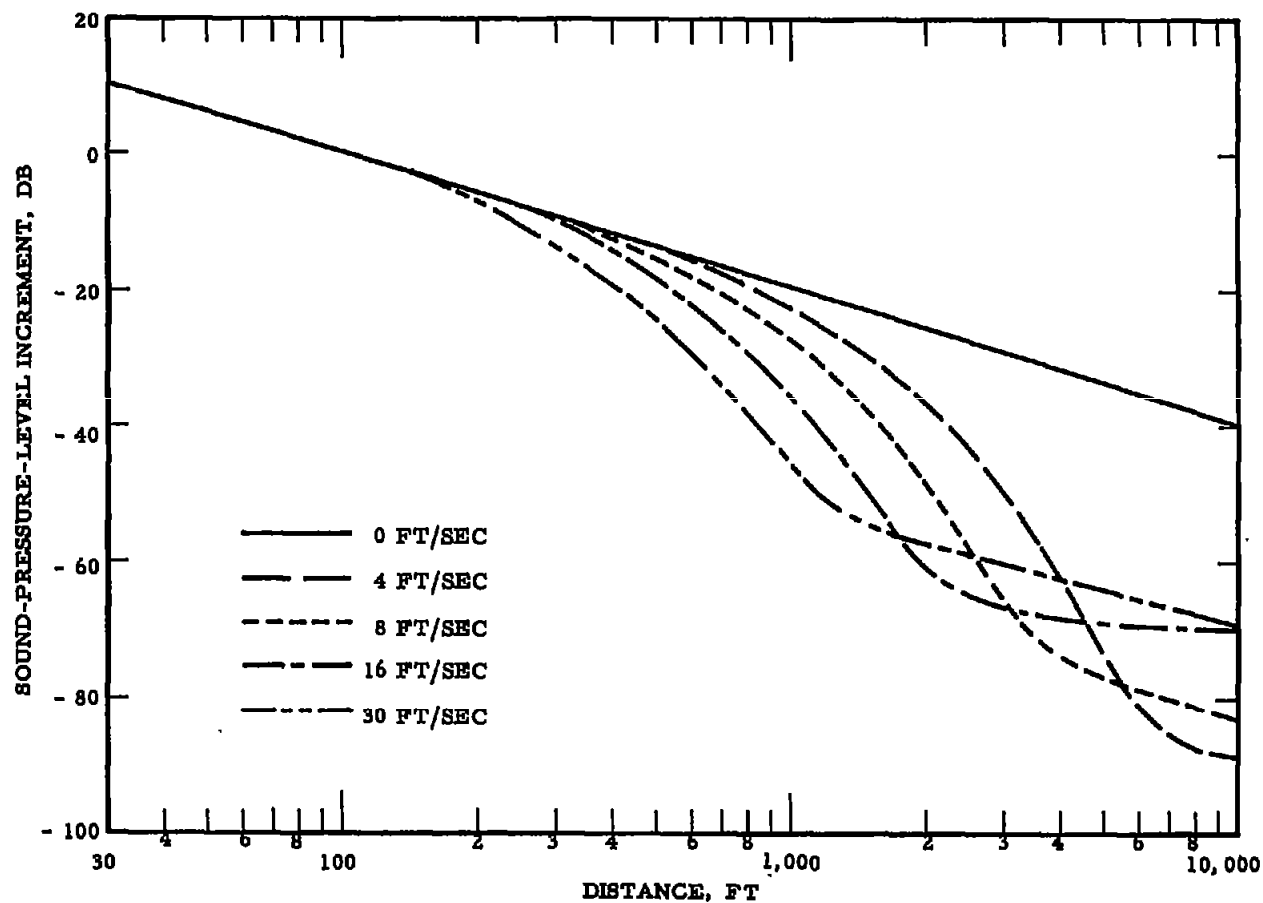
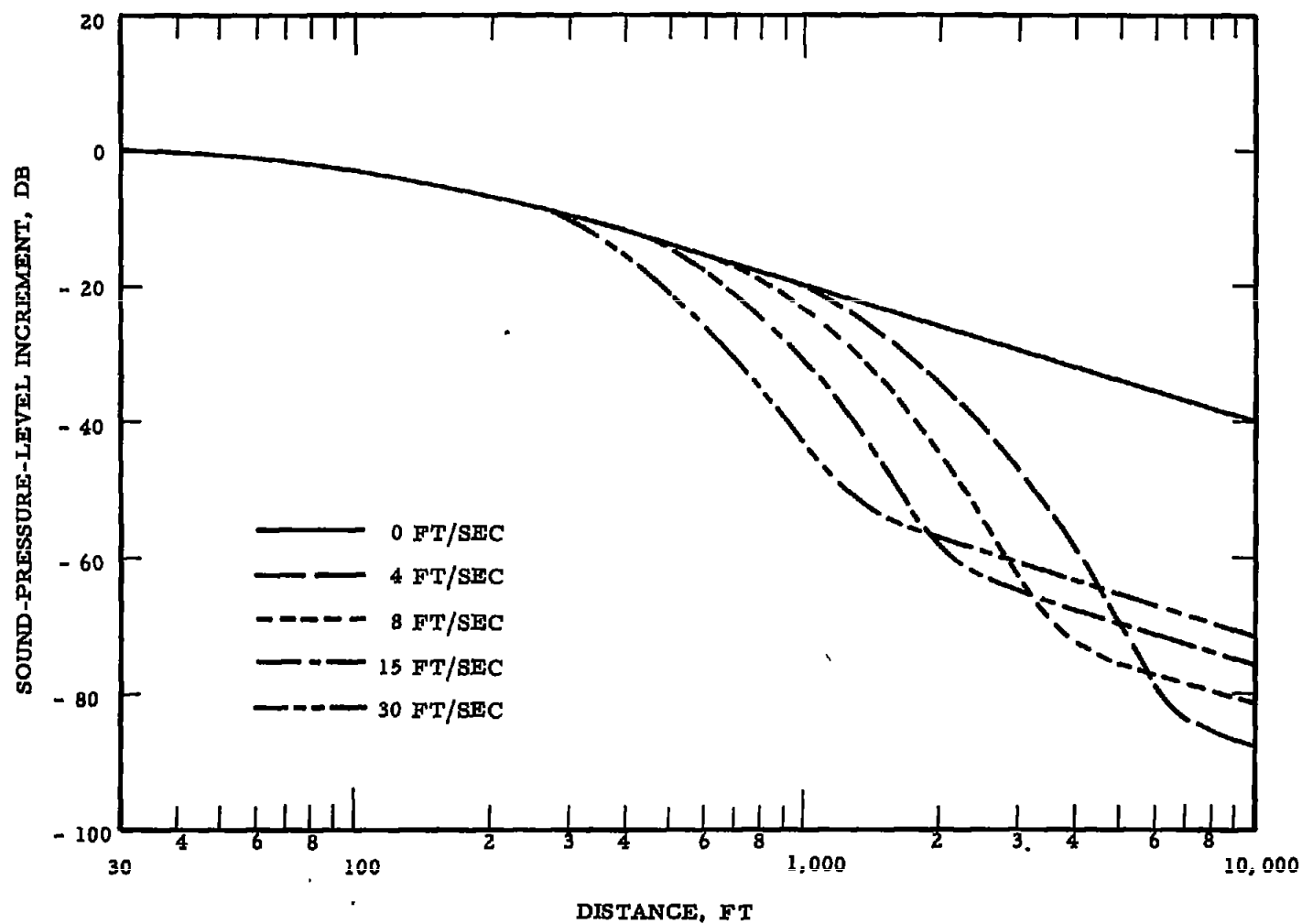


Figure 3.- Plots of critical frequency as function of wind velocity for several values of source-to-receiver parameter r and for source height of 10 feet. Curves divide upper right-hand region where scattering predominates from lower left-hand region where diffraction predominates.



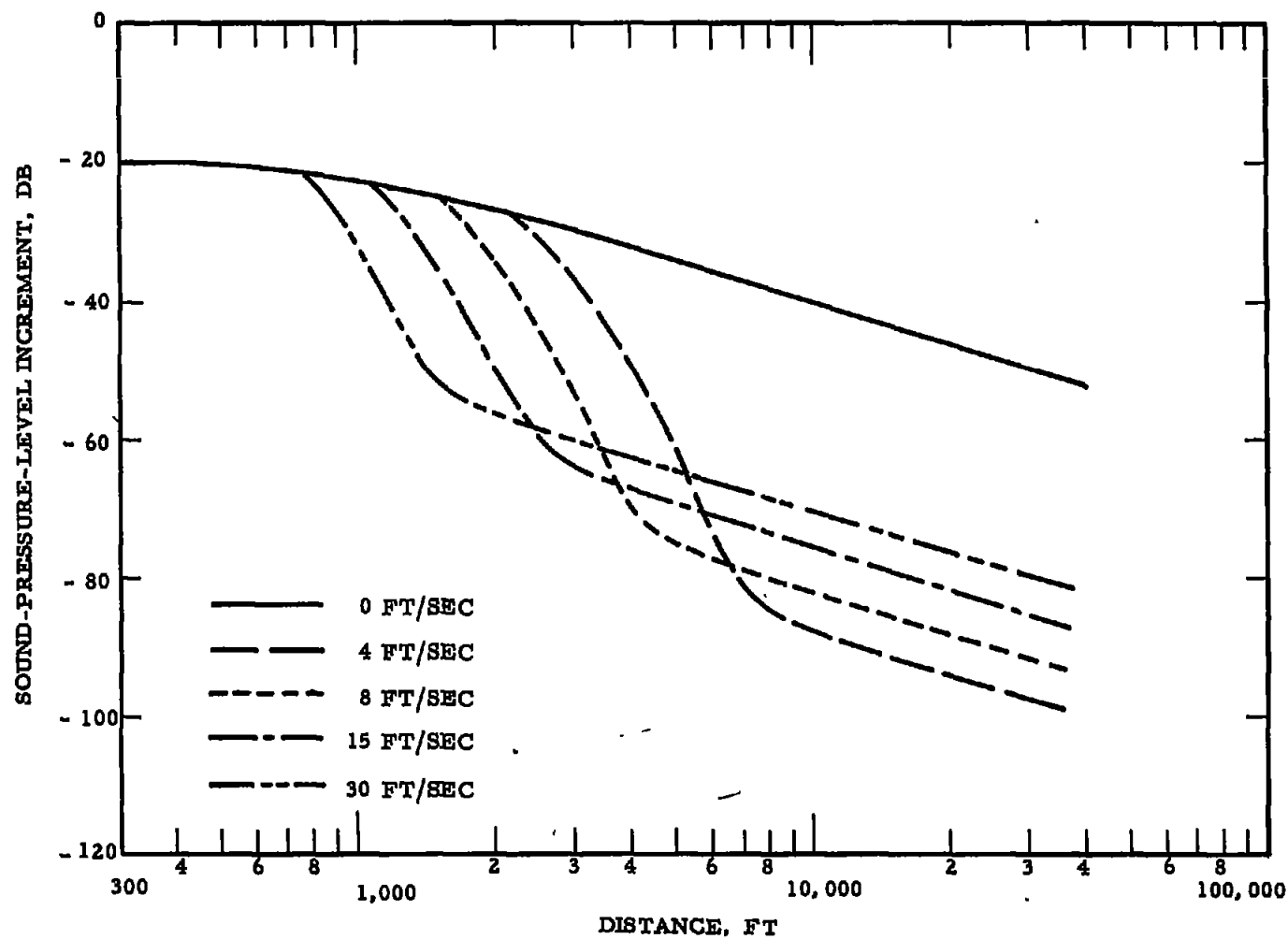
(a) Source height, 10 feet; frequency, 250 cps.

Figure 4.- Increment of sound pressure level as a function of distance from source for various wind velocities. Wind velocity is measured 10 feet above ground and is assumed to increase with height; ordinate gives increment of sound pressure level from reference sound pressure level as obtained in free field at 100 feet from source.



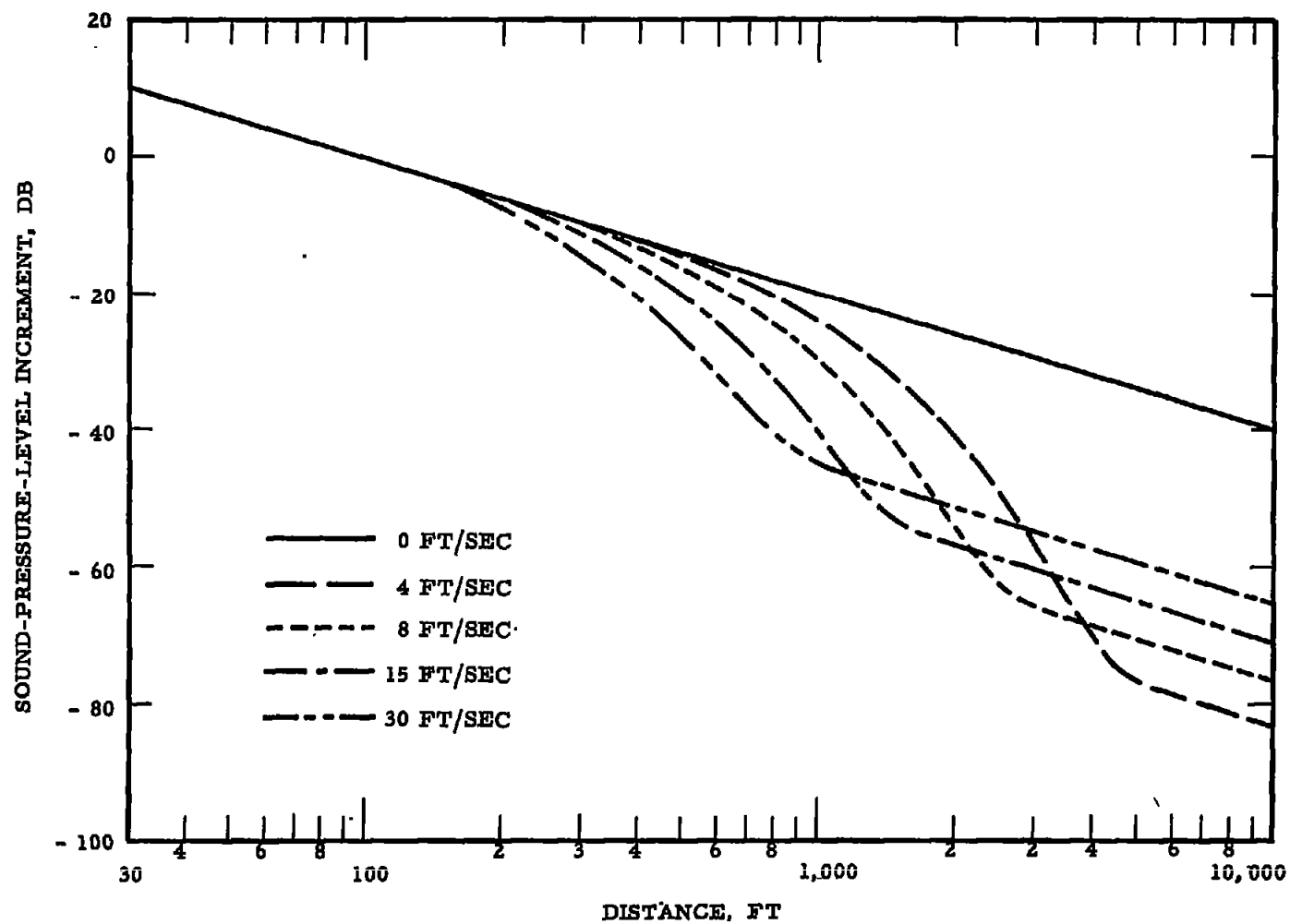
(b) Source height, 100 feet; frequency, 250 cps.

Figure 4.- Continued.



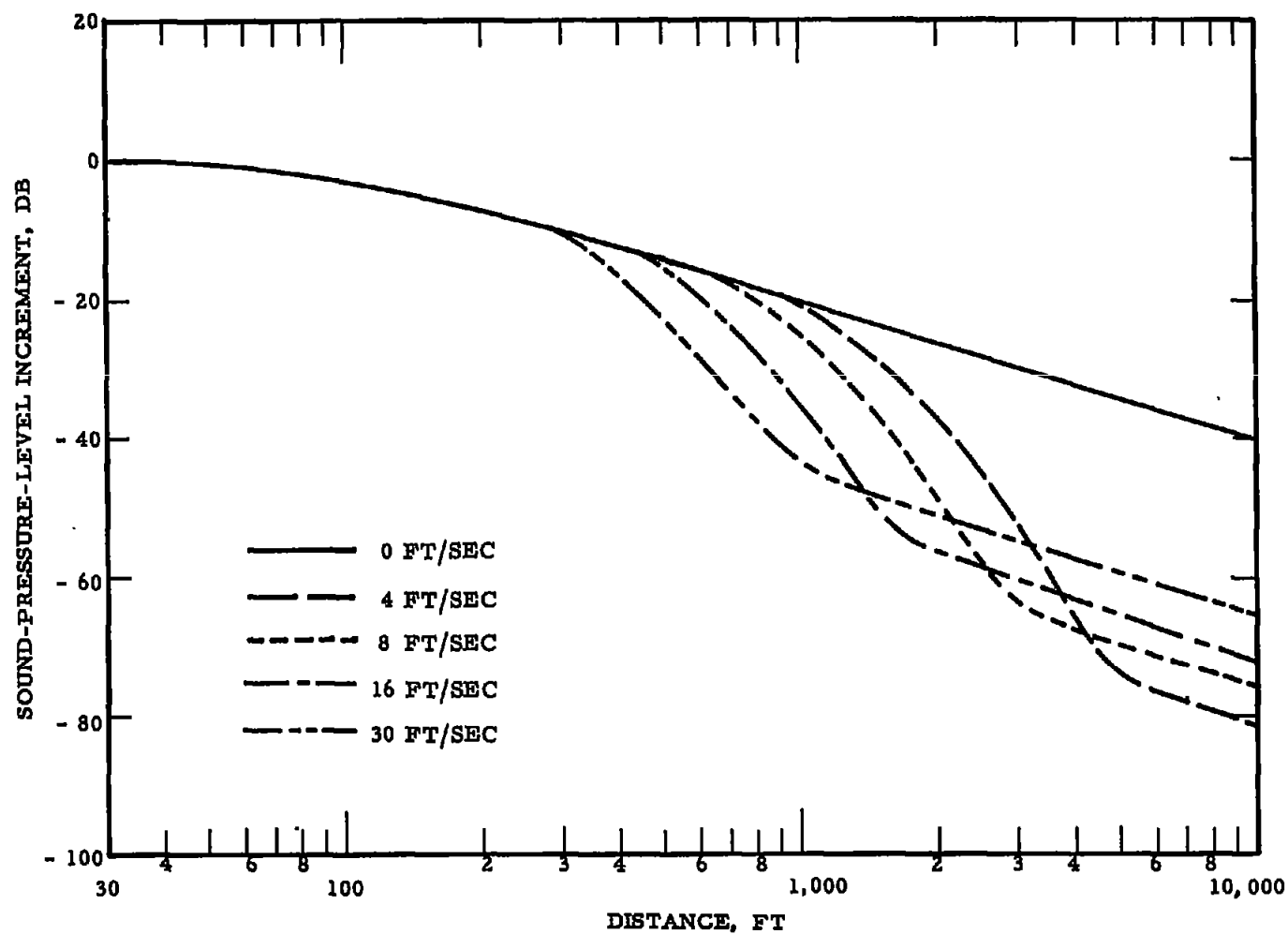
(c) Source height, 1,000 feet; frequency, 250 cps.

Figure 4.- Continued.



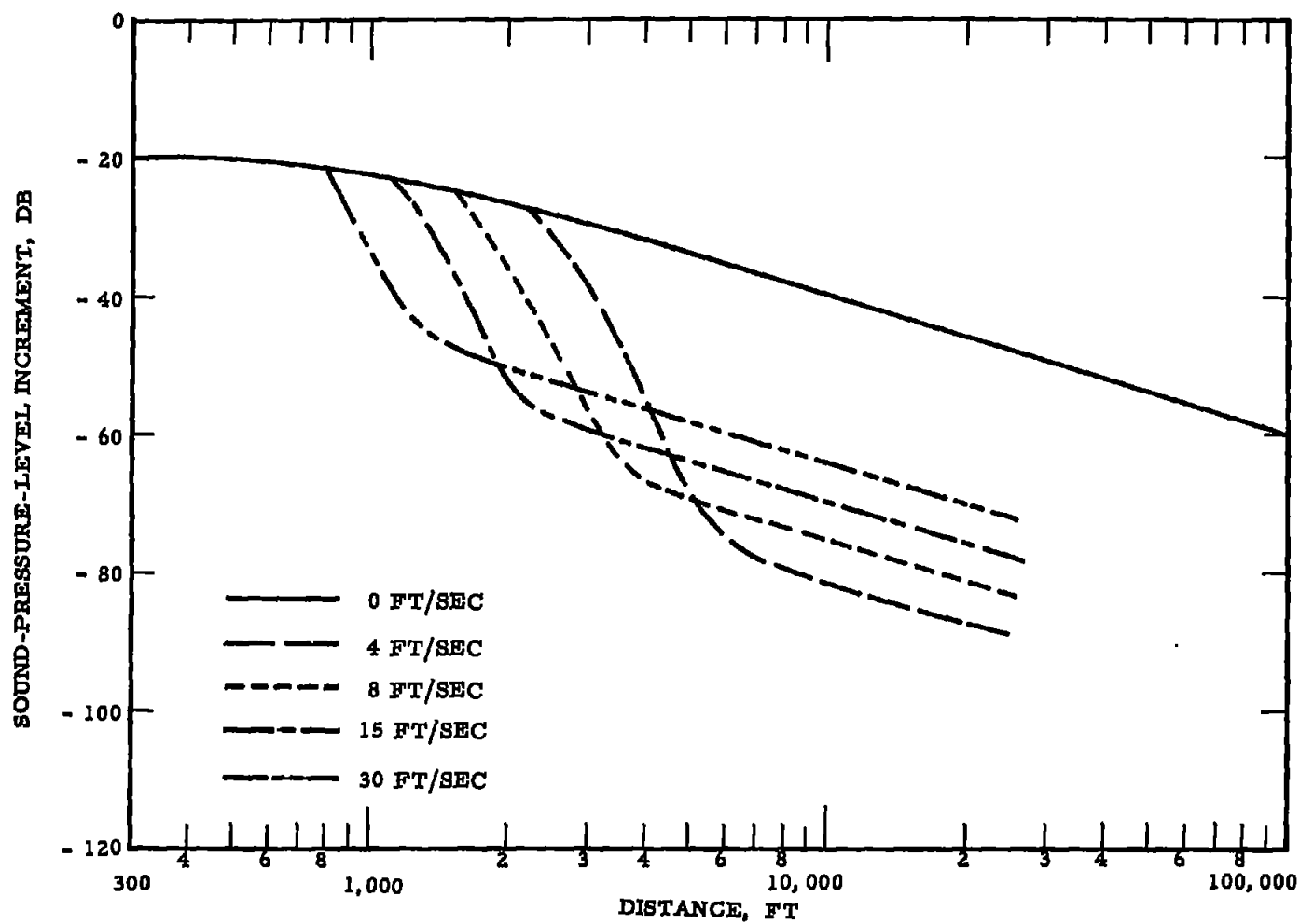
(d) Source height, 10 feet; frequency, 500 cps.

Figure 4.- Continued.



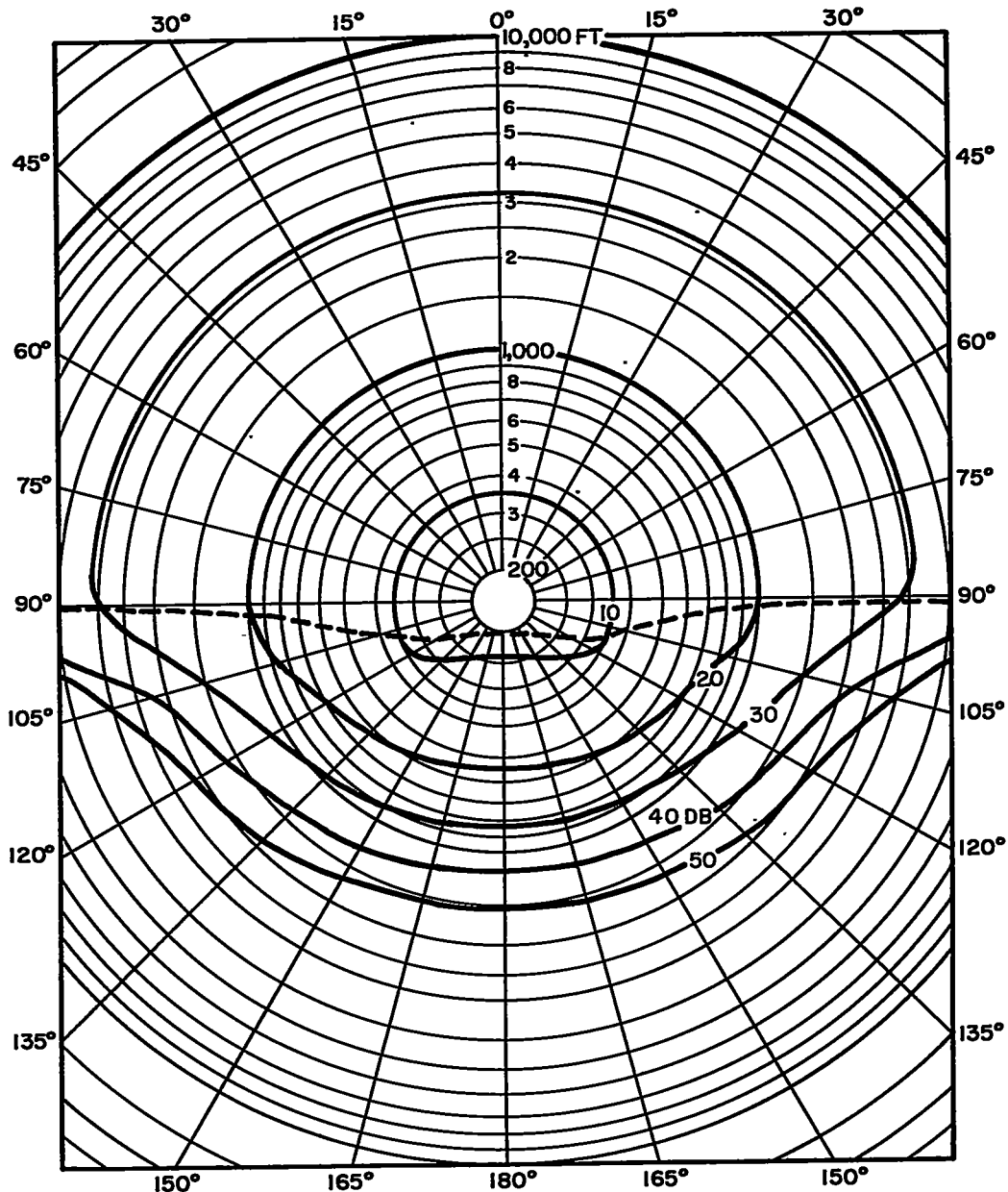
(e) Source height, 100 feet; frequency, 500 cps.

Figure 4.- Continued.



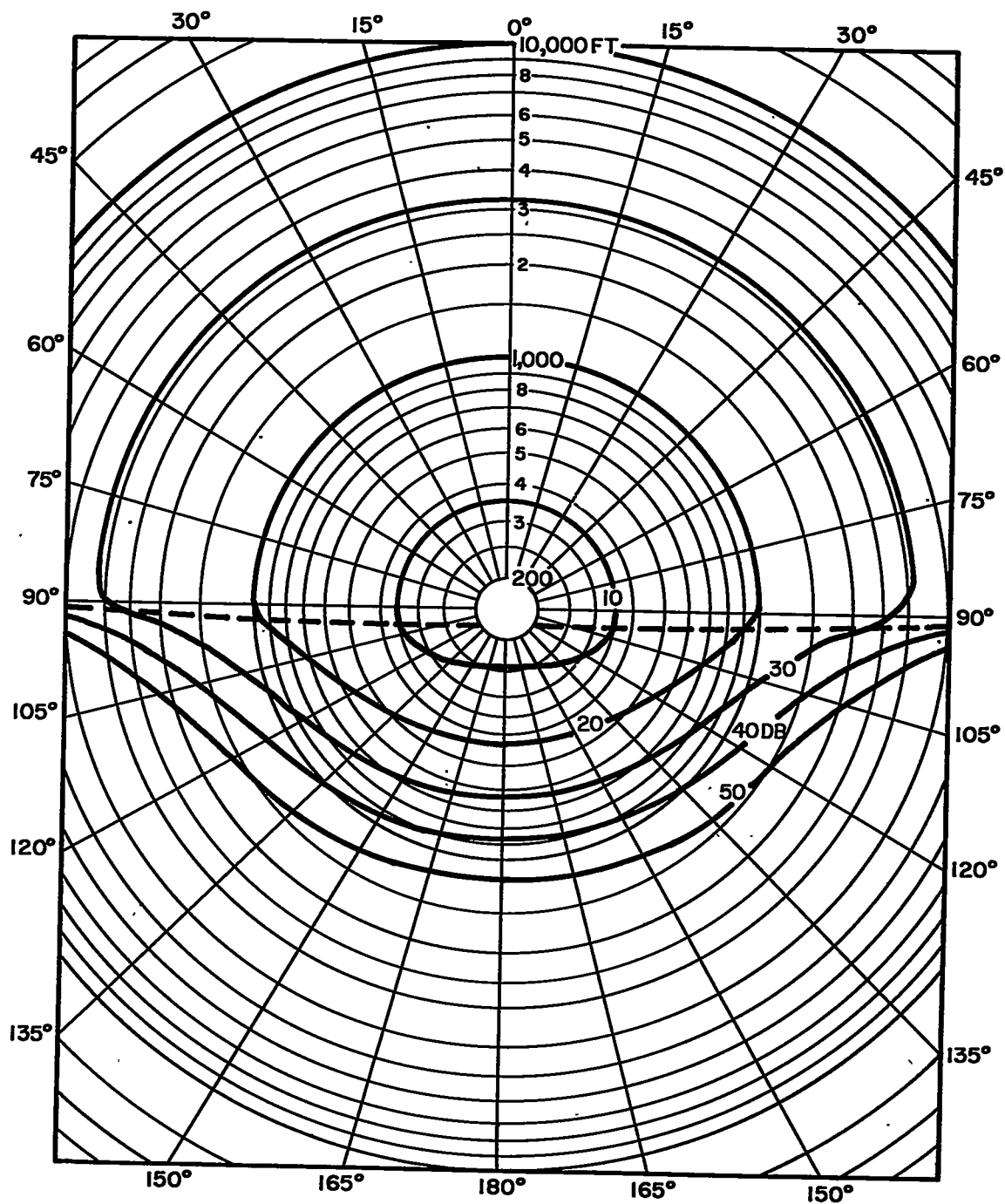
(f) Source height, 1,000 feet; frequency, 500 cps.

Figure 4.- Concluded.



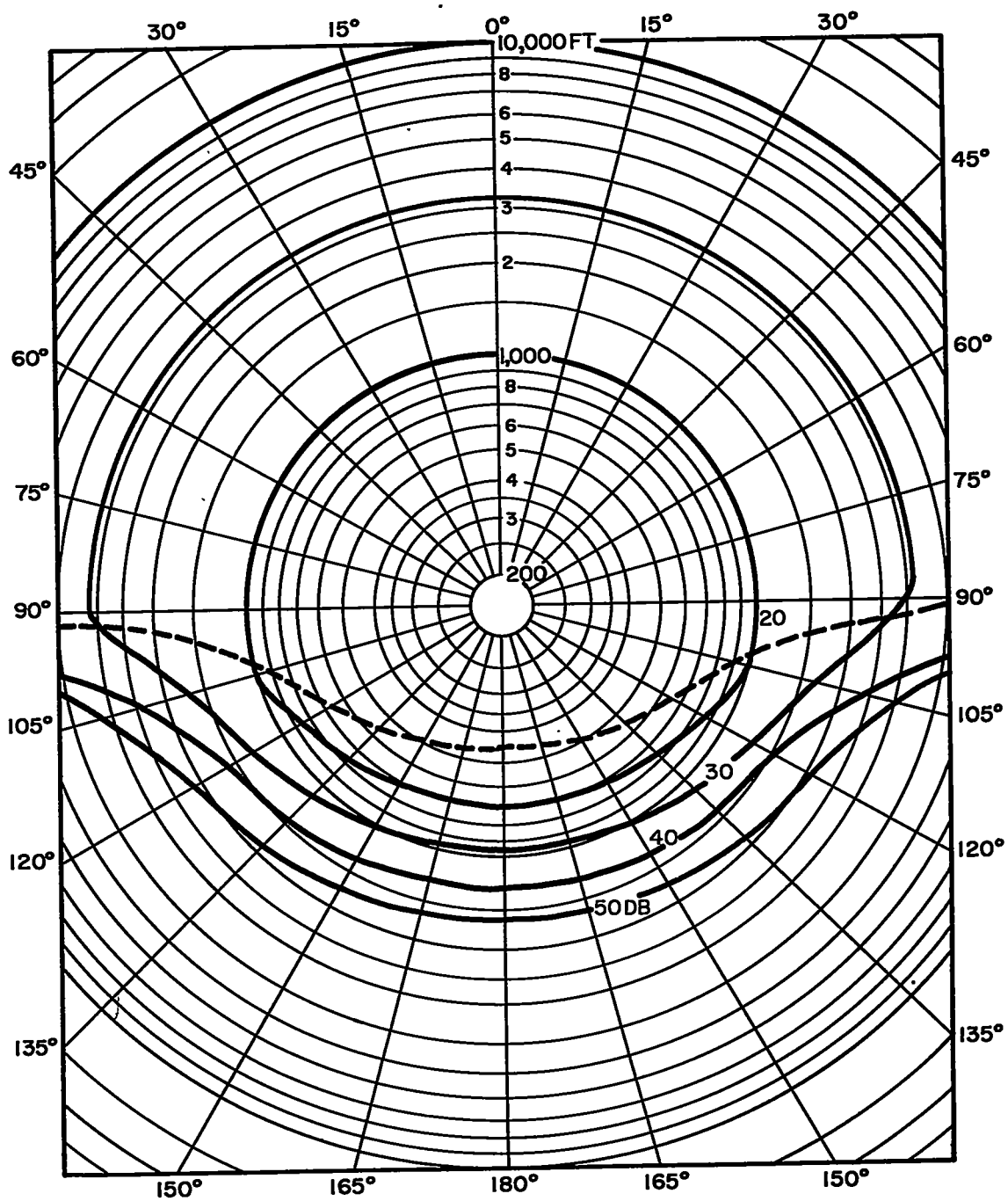
(a) Source height, 10 feet; wind velocity, 10 mph; frequency, 250 cps.

Figure 5.- Equal sound-level contours measured 10 feet above ground about source in wind for different wind velocities, source heights, and frequencies. Wind direction is toward 0°; decibel values on contours indicate decrease of sound pressure level below the reference sound pressure level as obtained in free field at 100 feet from source; dashed line indicates position of shadow boundary.



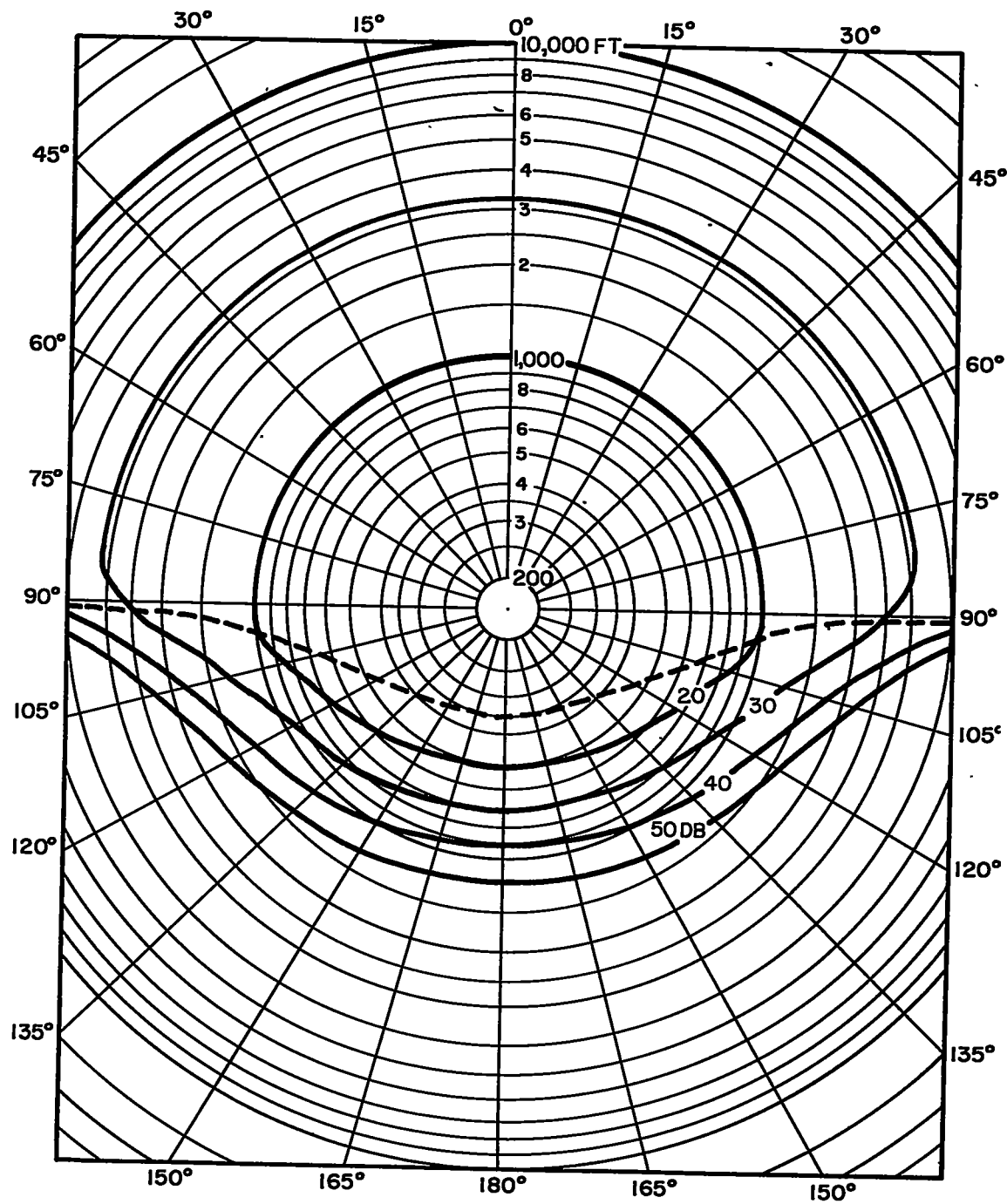
(b) Source height, 10 feet; wind velocity, 20 mph; frequency, 250 cps.

Figure 5.- Continued.



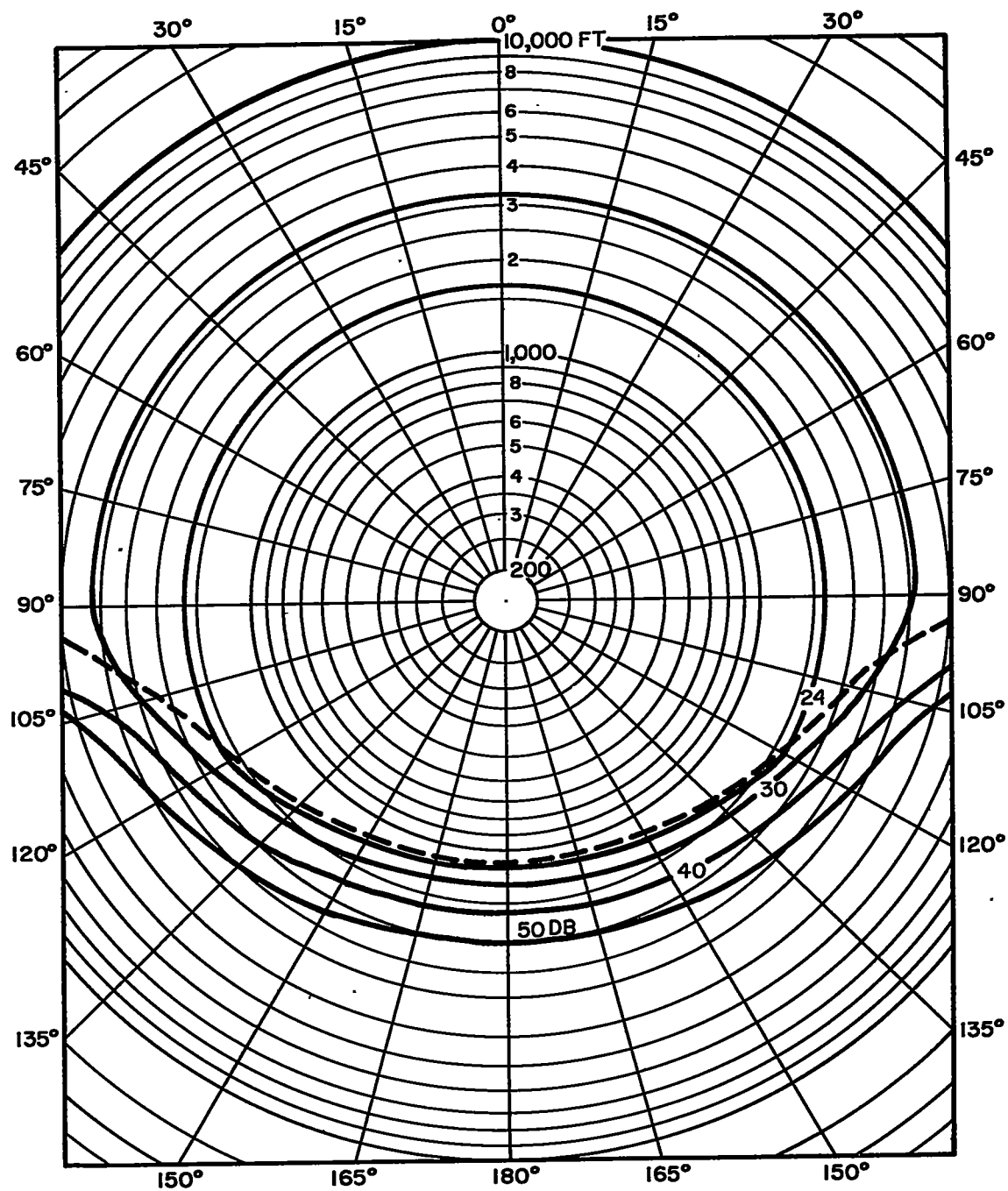
(c) Source height, 100 feet; wind velocity, 10 mph; frequency, 250 cps.

Figure 5.- Continued.



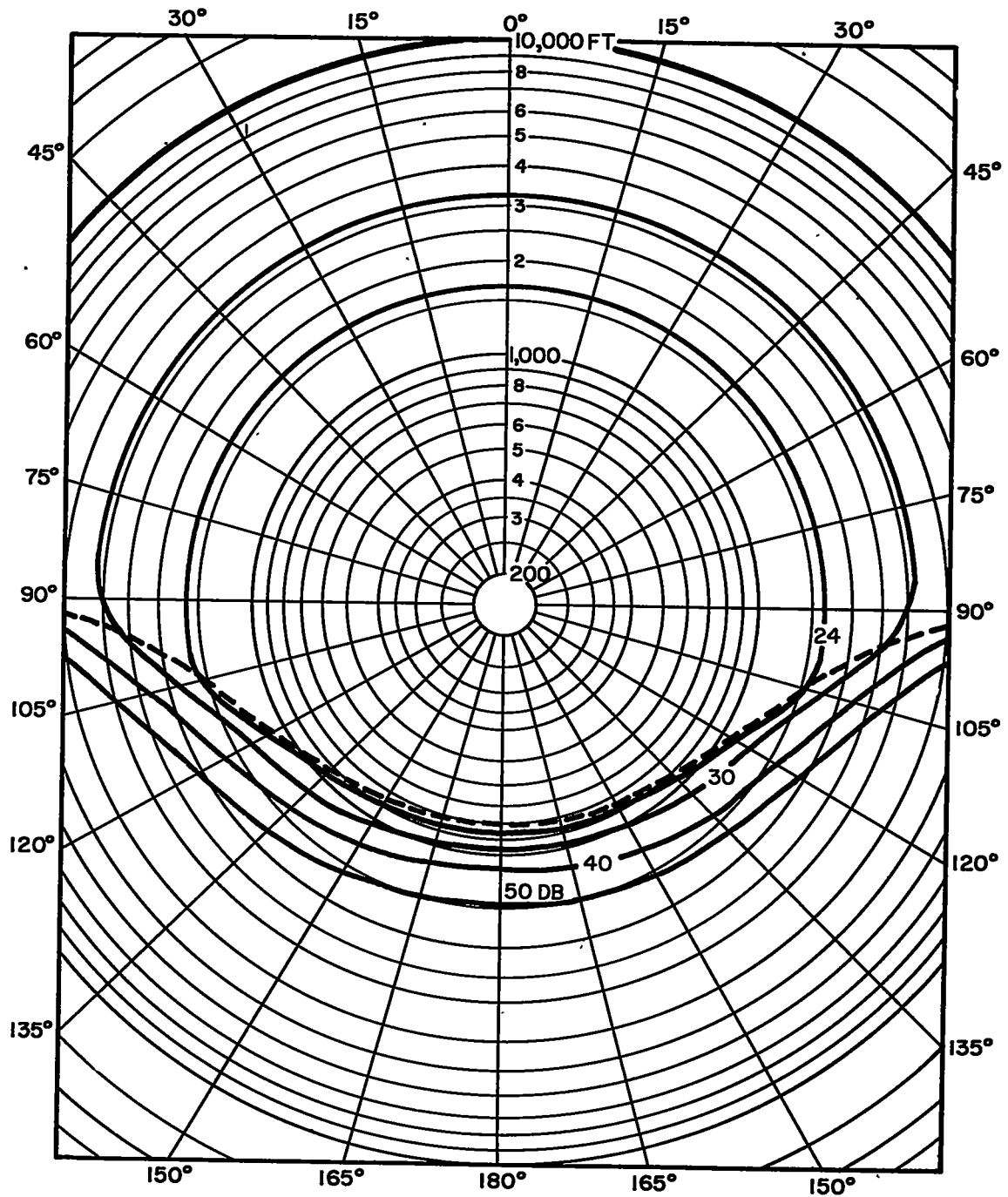
(d) Source height, 100 feet; wind velocity, 20 mph; frequency, 250 cps.

Figure 5.- Continued.



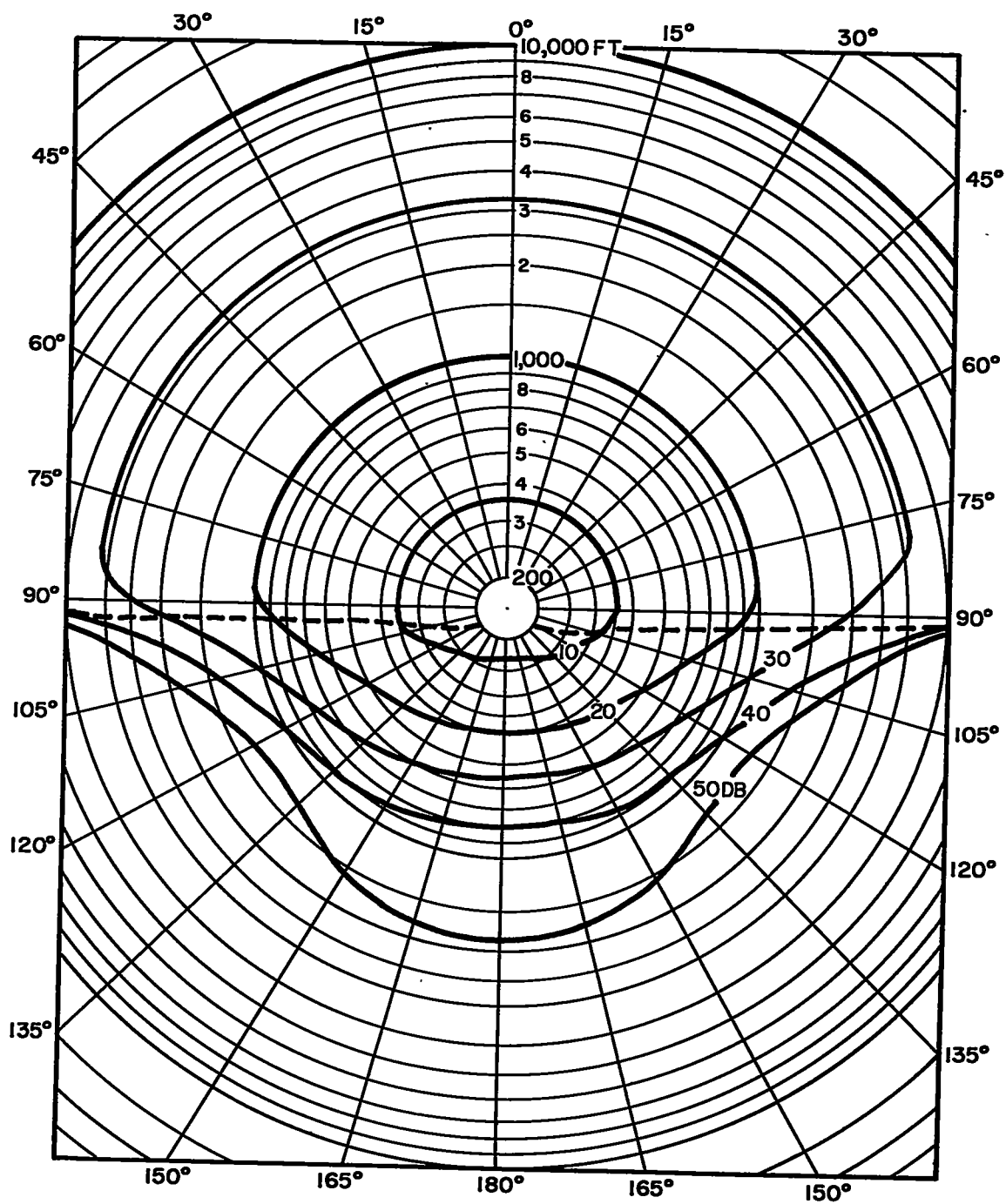
(e) Source height, 1,000 feet; wind velocity, 10 mph; frequency, 250 cps.

Figure 5.- Continued.



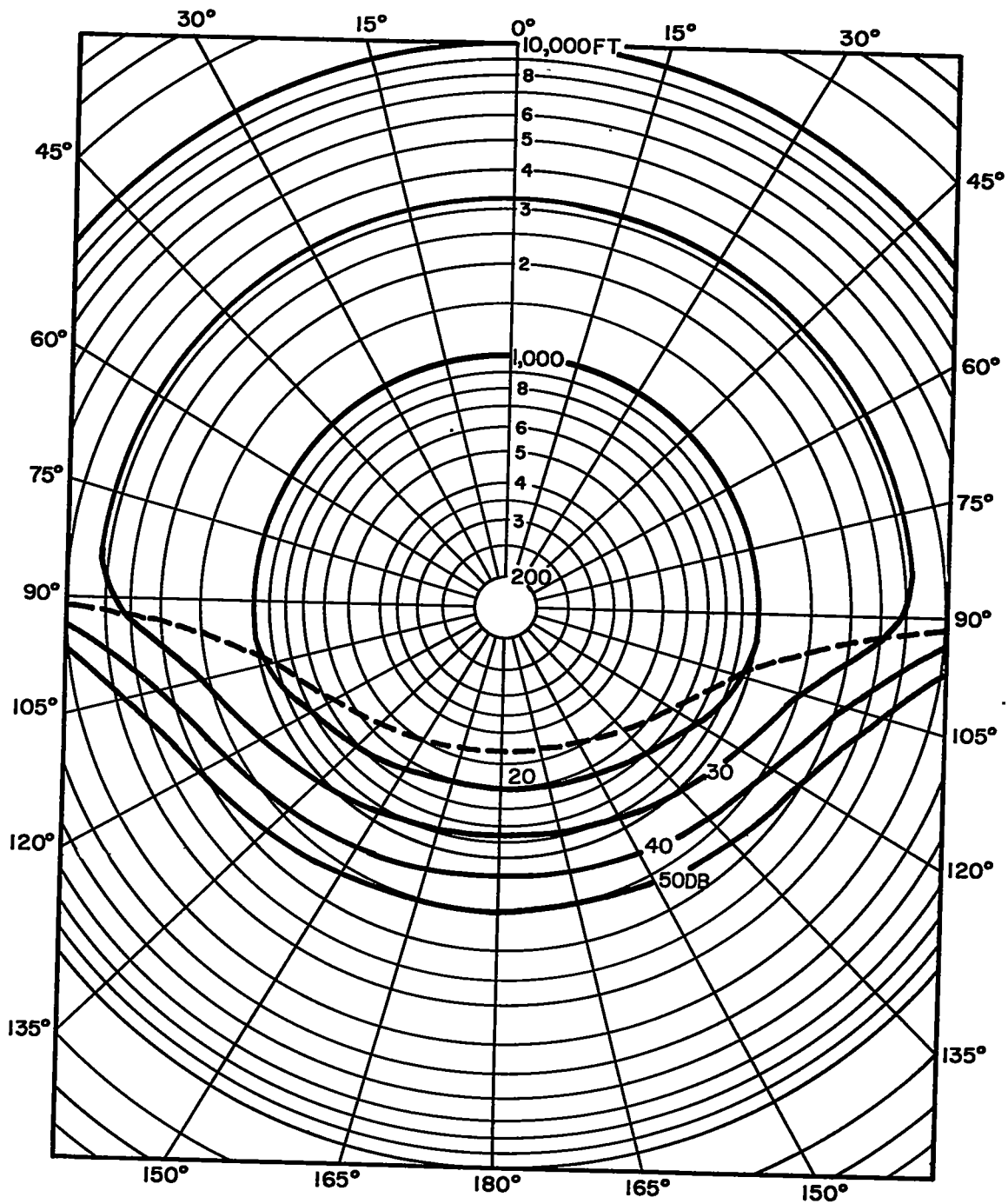
(f) Source height, 1,000 feet; wind velocity, 20 mph; frequency, 250 cps.

Figure 5.- Continued.



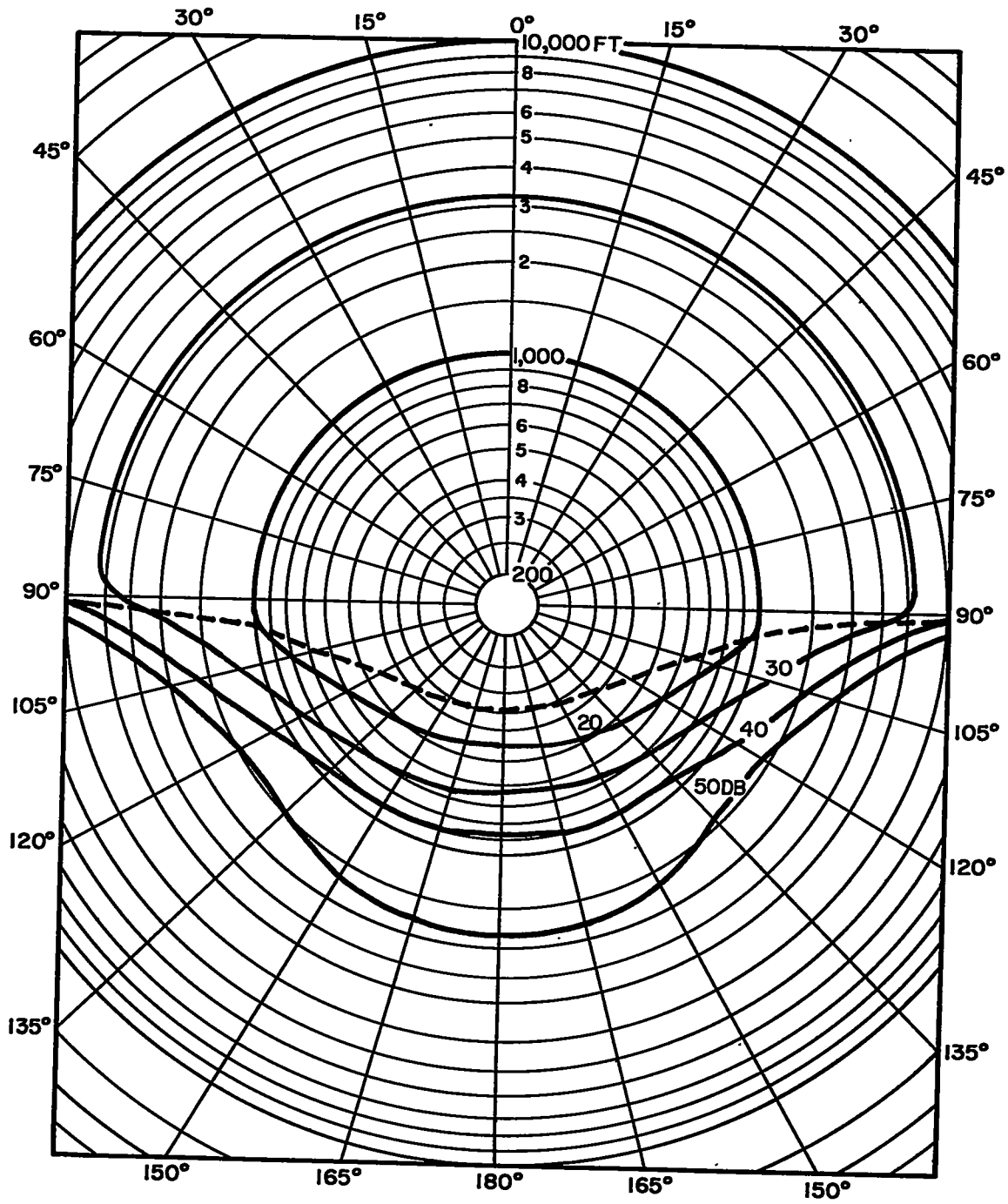
(h) Source height, 10 feet; wind velocity, 20 mph; frequency, 500 cps.

Figure 5.- Continued.



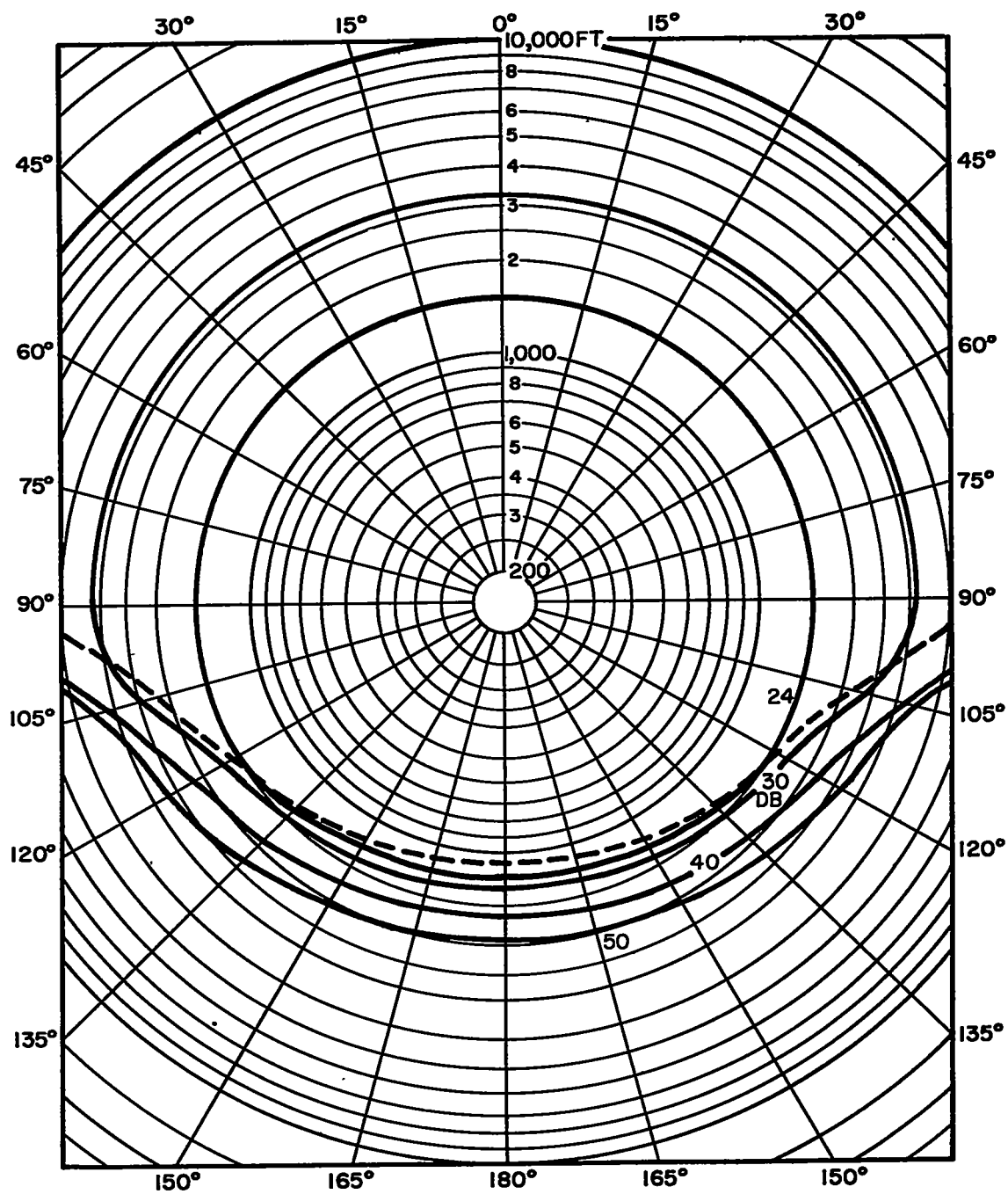
(i) Source height, 100 feet; wind velocity, 10 mph; frequency, 500 cps.

Figure 5.- Continued.



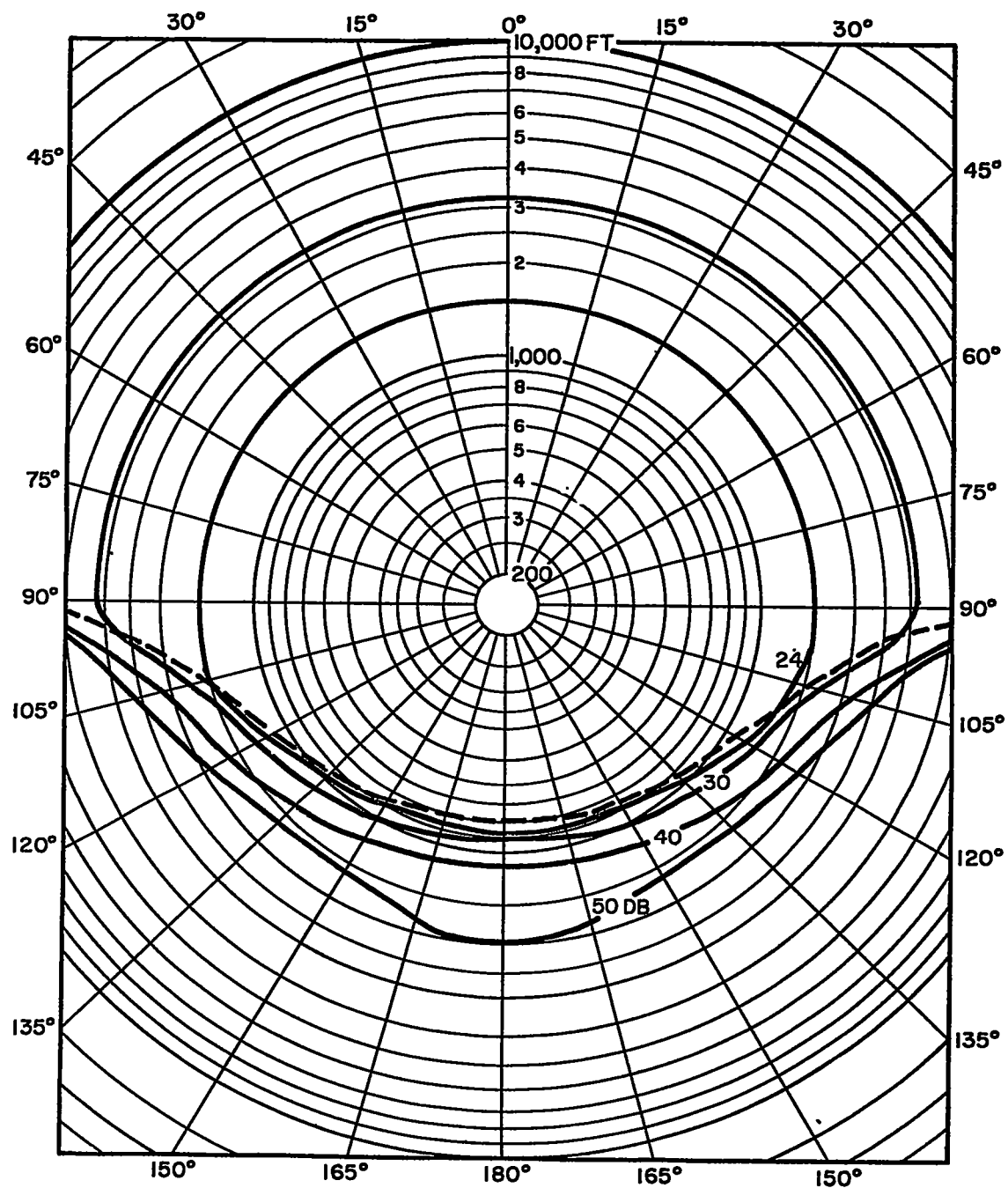
(j) Source height, 100 feet; wind velocity, 20 mph; frequency, 500 cps.

Figure 5.- Continued.



(k) Source height, 1,000 feet; wind velocity, 10 mph; frequency, 500 cps.

Figure 5.- Continued.



(2) Source height, 1,000 feet; wind velocity, 20 mph; frequency, 500 cps.

Figure 5.- Concluded.